EECS C145B / BioE C165 Spring 2003: Problem Set VI (Practice final) WILL NOT BE GRADED

Total points: 770 + 110 bonus.

Question 1 (20+10 bonus)

You are given a projection matrix $\mathbf{F}$ for a tomographic system that has a spatially invariant point spread function.

1. How can you find this PSF from $\mathbf{F}$?

2. * How can you find this PSF directly from $\mathbf{F}^T\mathbf{F}$ without taking a matrix square root?

Question 2 $(10 + 20)$

You rapidly inject 1 mCi of ^{131}I-albumin into a major artery at $t = 0$ seconds. This tracer does not "stick" in the heart muscle and appears in the myocardium as $v_1(t) = 5\,\mathrm{e}^{-(t-4)/2}$ Bq. You simultaneously inject 1.2 mCi of $^{99\mathrm{m}}$Tc-sestamibi. This tracer is preferentially retained in the myocardium.

1. Can you measure the time activity curve of $^{99\mathrm{m}}$Tc-sestamibi in the myocardium $v_2(t)$ independently of $v_1(t)$? Explain.

2. In a published paper where the myocardium is modeled using a linear single compartment model, the wash-out parameter of the myocardial compartment is estimated at $k_2 = 1/60$ min^{-1} for $^{99\mathrm{m}}$Tc-sestamibi. Estimate $v_2(t)$ and justify any assumptions.

Question 3 $((10 + 10 + 10 + 10 + 10))$

Explain whether LSO is superior to BGO in terms of the following physical parameters and discuss the impact on scanner design and performance:

1. LSO attenuation length = 1.2 cm, BGO attenuation length = 1.1 cm.

2. LSO photons/MeV $= 25,000$, BGO photons/MeV $= 8,200$.

3. LSO decay time $= 40$ns, BGO decay time $= 300$ns.

4. LSO emission wavelength $= 415$nm, BGO emission wavelength $= 480$nm.

5. LSO intrinsic decays per second $= 300$, BGO intrinsic decays per second $= 0$.

Question 4 $((20 + 20)$

1. Show by a flow diagram the method of arithmetic reconstruction.

2. Show how you would find a solution to the inverse problem below (This entails finding the values for the pixels of the image a_i, given the projections p_k).

$$\mathbf{p} = \mathbf{F}\,\mathbf{a}$$

a_1	a_2	$p_1 = 4$
a_3	a_4	$p_2 = 7$
$p_4 = 8$	$p_3 = 5$	

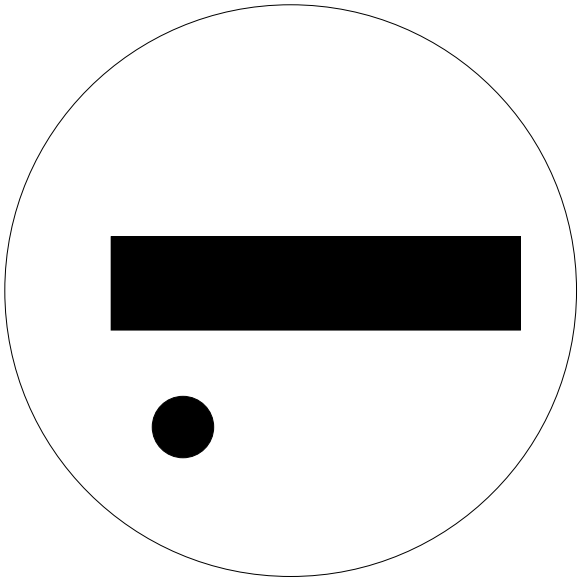
Question 5 (60)

Fill in the following table:

	Type of signal	**Property(ies) which modifies(y) signal**
Chest x-ray or mammography		
X-ray CT		
PET or SPECT		
MRI		
Ultrasound		
Optical imaging		

Question 6 $((15 + 15))$

Projection MRI imaging is to be performed on the object below. Angles are measured anticlockwise from the horizontal.



Show the projection obtained when a gradient field is applied at 30 degrees.

Show the projection obtained when a gradient field is applied at 180 degrees.

Question 7 ((60))

With the assistance of diagrams, explain how you would perform 2D Fourier transform MRI of an image of $M \times M$ pixels is required. You explanation should include both data acquisition and reconstruction methods. Assume that a simple spin-warp sequence is used to obtain the acquired signal.

Question 8 $((10 + 10 + 10))$

An image is acquired using a spin-warp imaging sequence, but the values of TR and TE used have been forgotten. Given the information in figure 1, determine whether TR and TE were long or short, relative to T1 and T2, respectively, when the image in figure 2 was acquired.

Check one box:

		Answer
TR short	TE long	
TR long	TE short	
TR long	TE long	
TR short	TE short	

Suppose the fourth blob was the brightest blob. What would you say about TR and TE?

Check one box:

		Answer
TR short	TE long	
TR long	TE short	
TR long	TE long	
TR short	TE short	

Suppose the first and second blobs were bright and the third and fourth dark. What would you say about TR and TE?

Check one box:

		Answer
TR short	TE long	
TR long	TE short	
TR long	TE long	
TR short	TE short	

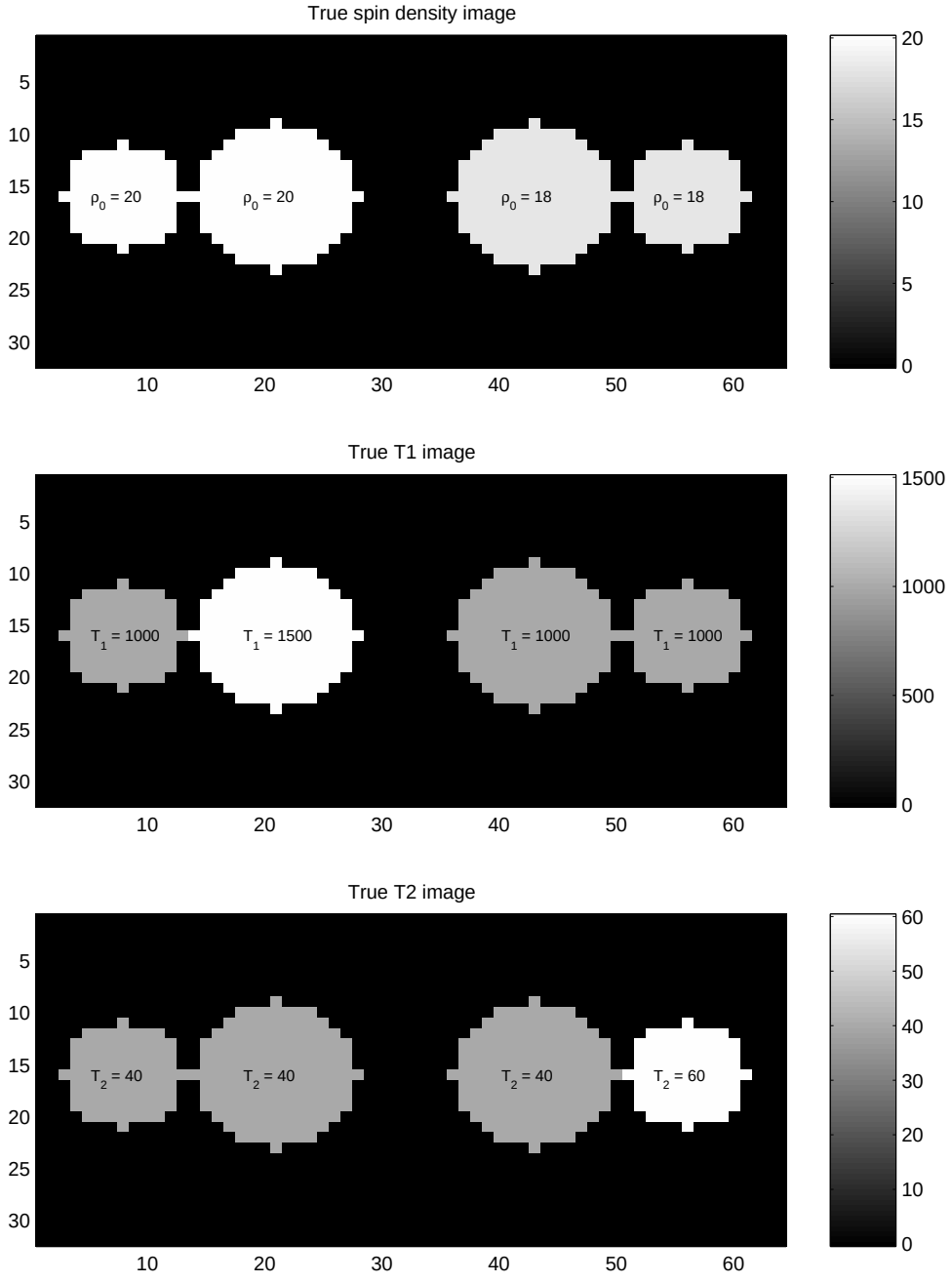


Figure 1: True T1, T2 and spin-density (ρ_0) distributions

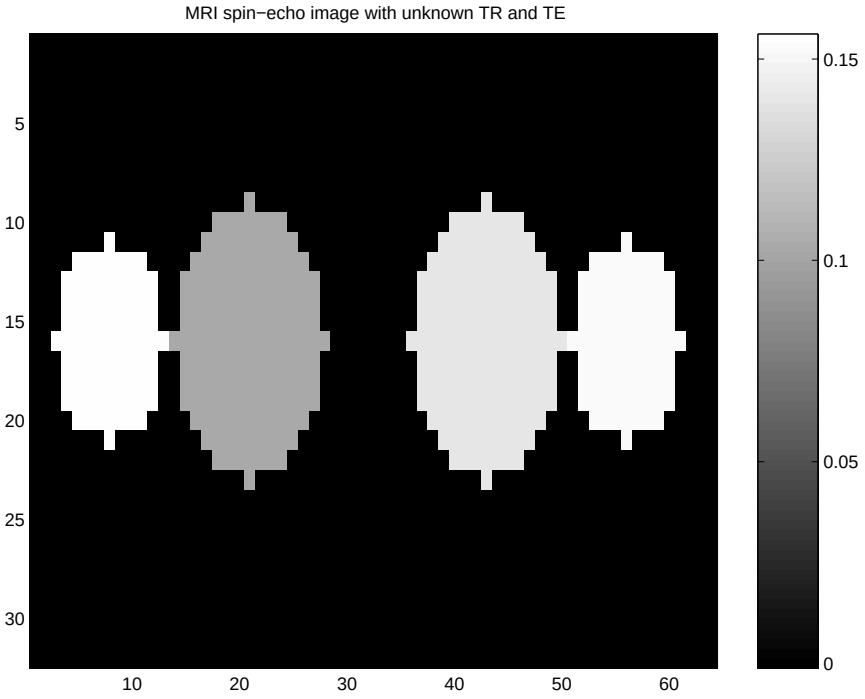
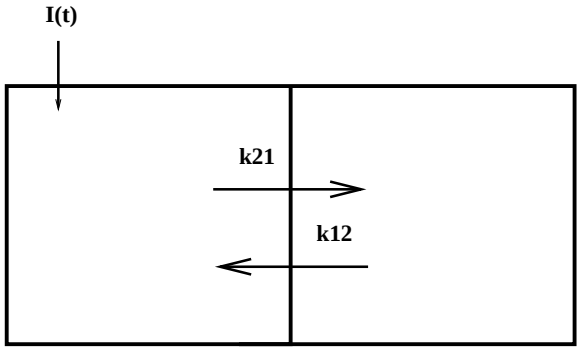


Figure 2: Image obtained with TE and TR unknown

Question 9 ($(25 + 15) + 15$ bonus)

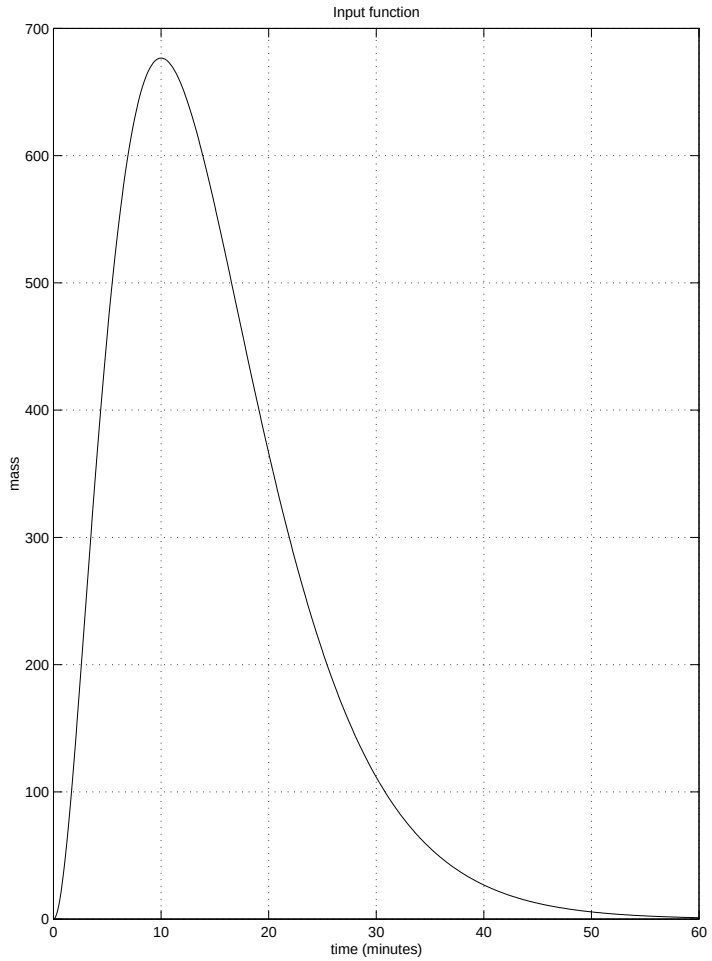
For the following compartmental model,



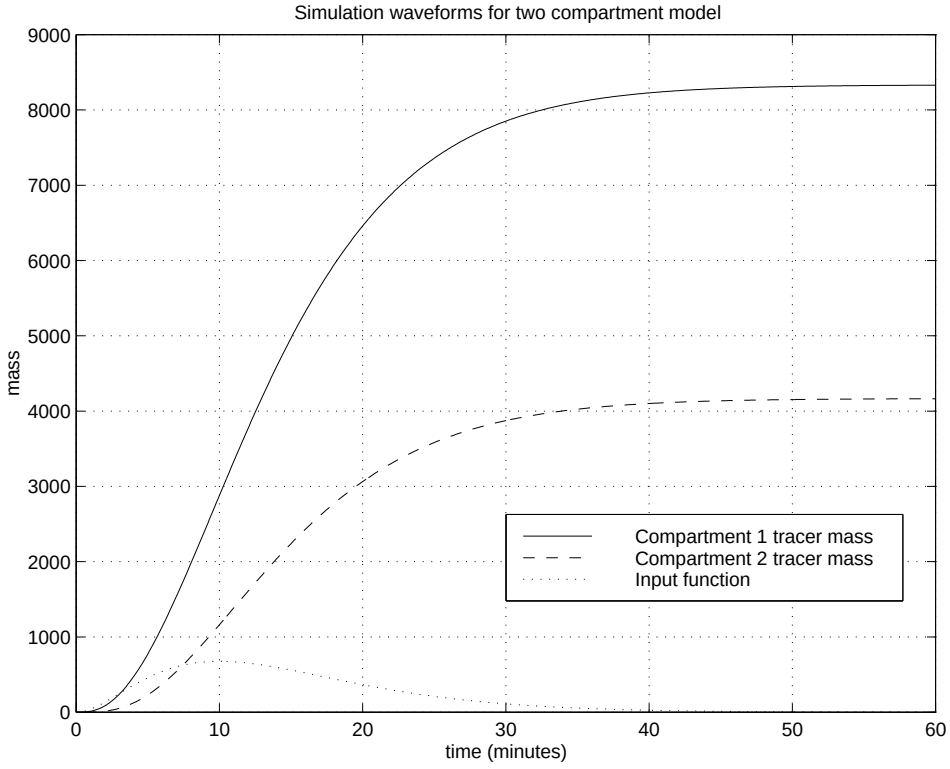
1. Write the differential equations which describe this system in terms of the compartment sizes q_1 and q_2, and the fractional transfer coefficients k_{12} and k_{21}.

2. Derive an expression for the ratio $\frac{k_{12}}{k_{21}}$ which does not involve any derivative terms. Explain the significance of this result for static as opposed to dynamic emission tomography.

3. The system was excited using this input function:



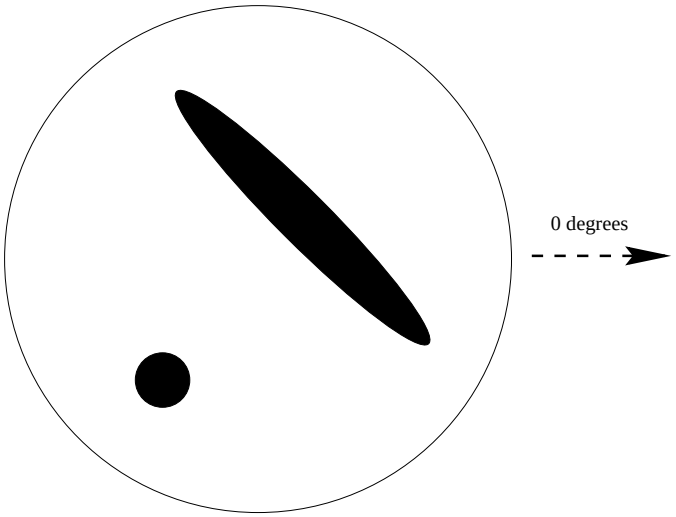
and the following output was obtained:



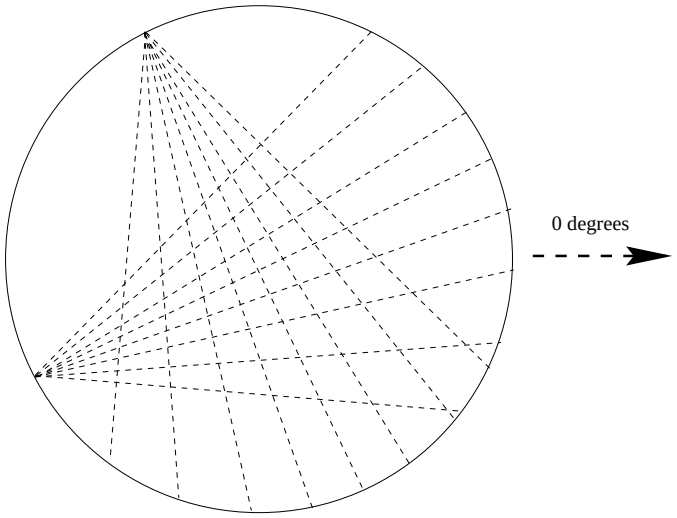
Estimate the fractional transfer coefficients.

Question 10 ($(20) + 20$ bonus)

1. Draw the sinogram of the objects in the distribution below. Do not worry about the intensity, just draw the boundaries of the sinograms of both objects.



2. Show the sampling of the sinogram for the projection geometry (lines of repsonse) shown below. Draw small circles on the sinogram to show where the samples occur.



Question 11 (20)

Derive a formula that gives the minimum number of angular projections needed in SPECT for a circular object n resolution elements wide.

Draw a diagram to illustrate your proof.

Question 12 $(20 + 20)$

Show how you would find a solution to the inverse problem below (This entails finding the values for the pixels of the image a_i, given the projections p_k).

$$\mathbf{p} = \mathbf{F}\,\mathbf{a}$$

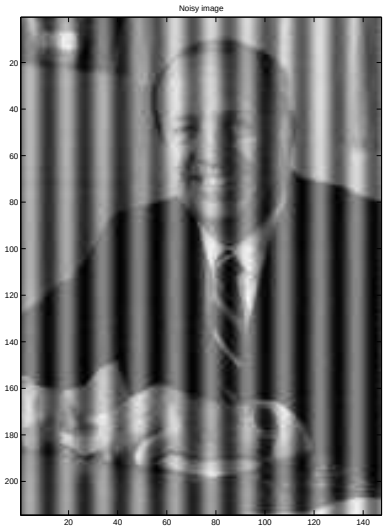
a_1	a_2	$p_1 = 4$
a_3		$p_2 = 7$
a_4		$p_3 = 2$
	a_5	$p_4 = 2$
$p_5 = 8$	$p_6 = 5$	

1. Using the algebraic reconstruction technique (ART) (show one iteration).

2. Using the weighted least-squares method. (formulate matrices and matrix equation but do not solve). Assume projection bin variances are equal to projection bin values.

Question 13 (30)

Show diagramatically how to remove the artifact in the image below.



1. Using a digital methods:

2. Using an analog method:

Question 14 $(10 + 20 + 10)$

Values of T1 and T2 for brain are:

	T1 (ms)	T2 (ms)
CSF	3000	40
gray	1100	30
white	800	35
tumour	1200	80

You are able to vary TR and TE, such that any combination of the following is allowed:

TR	3000, 500 ms
TE	30, 80 ms

1. What is the optimal combination for imaging gray matter-white matter contrast?

2. What is the optimal combination for delineation of the CSF space?

3. Show a practical full pulse sequence for tumor detection.

Question 15 (30)

You are designing a small animal SPECT system. What type of collimator would you use to:

1. Optimize resolution and sensitivity when the detector area is similar in size to the animal?

   ---

2. Optimize resolution and cost when the detector area is intended for human imaging?

   ---

3. Optimize resolution and field of view when the detector area is intended for human imaging?

   ---

4. Optimize resolution and sensitivity when the detector area is intended for human imaging?

   ---

5. Optimize field of view when only a small region of the detector area is functional?

Question 16 ($15 + 10 + 15$ bonus)

You are designing a B-mode ultrasound system for imaging the prostate gland.

1. The preliminary images from the instrument are disappointing because only some of the walls of the gland are visible. You would not be able to identify all types of tumor from these images. What is going wrong, and how could you modify the electronics and/or software to improve the images?

2. A former employee designed a linear probe for the system. Should you propose a design change? Why?

3. The system is intended to be used to locate the prostate so that simultaneous PET imaging can be performed. A consultant to your company is worried that the rectal probe will produce severe PET image artifacts. Is this a problem? Explain.

Question 17 $((20 + 20 + 20))$

1. During systole, an artificial heart pumps blood from rest at a constant acceleration of 2 m/s^2 for 0.5 seconds. Draw the CW ultrasound signal of the resulting flow and the sonogram of this waveform. Assume an insonation frequency of 1.5 MHz and an angle of 60 degrees between the incident radiation and the flow. Scale and label all axes.

2. Above we assumed all the blood travels at the maximum velocity. At low speeds this not a good assumption. Draw blood velocity profiles for laminar and turbulent flow and the corresponding sonogram columns, plotted as power spectra.

Question 18 (30 + 20 bonus)

You are seeking venture capital funding for a new private hospital that will provide comprehensive health services and house part of the research division of a pharmaceutical company. Make a wishlist for imaging instruments and associated support equipment and justify the need for each.

Question 19 (20 bonus)

Many humans throughout the world, particularly pregnant women, eat soil. This is known as geophagia. Eating soil can lead to iron deficiencies and heavy metal poisoning. Can imaging be used to diagnose and/or help us understand the causes of this phenomenon? How? What are the limitations?

Question 20 (30)

Explain why PET resolution will never be better than approximately 1mm. Why is this theoretically not the case with SPECT?

Question 21 $(15 + 15)$

In performing steepest descent optimization on the function:

$$f(x, y) = x^2 + 3\ln(y)$$

we find ourselves at the point $(3, 3)$.

1. In what direction do we take our next step? (do not normalize to a unit vector)

2. Will a step size of 2 be acceptable? Explain.

Question 22 (20)

Explain in detail how natural paramagnetism and paramagnetic tracers may be employed in MRI to provide functional contrast.

Question 23 ((15 + 30) bonus)

The book "MRI basics" uses diagrams such as the one below to explain contrast in spin-echo images:

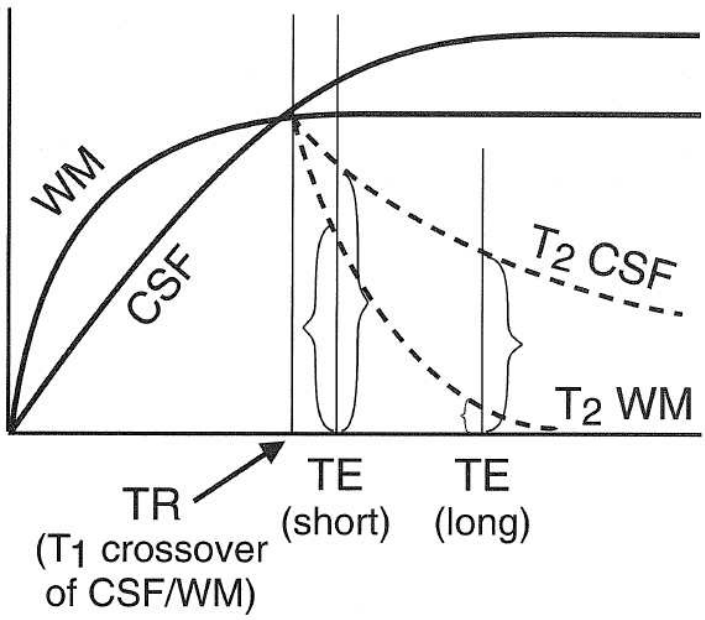


Figure 6-11. Recovery and decay curves of WM and CSF for a TR corresponding to the crossover point of CSF and WM.

Source: Hashemi p. 65

1. Why do you think the authors do not label the y-axis of the graph?

2. Explain the true physical reality with reference to the behavior of the field components M_{xy} and M_z, T_E and T_R, and contrast selection (Hint: Drawing two separate graphs might help resolve the issue in part 1 of this question).

.